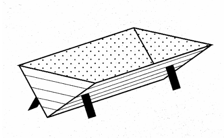
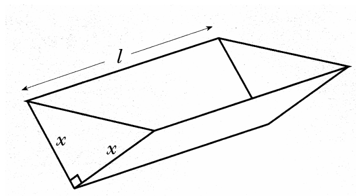
Banker Question: No. 12



An open water tank in the shape of a triangular prism, has a capacity of 108 litres.
The tank is to be lined on the inside in order to make it watertight.

The triangular cross section of the tank is right angled and isosceles with equal sides of length x cm.
The tank has length of l cm.



a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \dfrac{432000}{x}$ **(3)**

b) Find the value of x which minimizes this surface area. **(5)**

[Scroll to next page to see solution]

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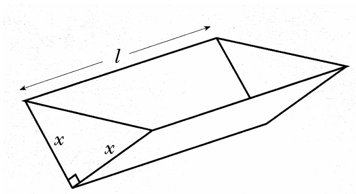
Solution.



a)

Area of an end triangle $= \frac{1}{2} \cdot x \cdot x$ (½ base × height)

Area of a side rectangle = $x \cdot l$

So Area, $A = 2\left(\frac{1}{2}x^2\right) + 2(xl)$

$A = x^2 + 2xl$

We are given that the volume is 108 litres.

Note the units. This is = 108 000 cm^3

The volume of a prism is $V = Ah$
where A is the cross section area.

So: $108000 = \frac{1}{2}x^2 \cdot l$ This is known as a constraint.

Rearranging for *l*:

$$l = \frac{216000}{x^2}$$

Substituting for *l* in our expression for *A*.

$$A = x^2 + 2x\left(\frac{216000}{x^2}\right)$$

This simplifies to:

$$A = x^2 + \frac{432000}{x}$$ as required.

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b)

Put into straight line form:

$$A = x^2 + 432000x^{-1}$$

Differentiate

$$\frac{dA}{dx} = 2x - 432000x^{-2}$$

For a SP, $\frac{dA}{dx} = 0$

$$2x - 432000x^{-2} = 0$$

Notes on solution

By looking at the question you should know immediately it is about differentiation and optimising – using constraints, stationary points, table of signs.

Find an expression for A, the surface area to be lined.

The area is composed of two triangles at the ends and the two rectangles for the sides.

We have two variables in this expression, we need to replace *l* with an expression in terms of *x*.

The volume given will link the length *l* to *x*.

Obtain an expression for *l* in terms of *x*.

Obtain an expression for *l* in terms of *x*.

Substitute and simplify.

If you cannot obtain this, then you can use the **given** answer to part *(a)* to gain the marks for part *(b)*

We want to find the value of *x* which minimises
the surface area.

We want the stationary points of the function.

To find these we differentiate.

Put back into positive index form:

$$2x - \frac{432000}{x^2} = 0$$

Rearrange

$$2x = \frac{432000}{x^2}$$

Cross multiply:

$$2x^3 = 432000$$

Divide by 2

$$x^3 = 216000$$

$$x = \sqrt[3]{216000} = 60$$

There is only one stationary point, $x = 60$

We now need to show this is a minimum using the table of signs.

	1		100
x	$\rightarrow$	60	$\rightarrow$
$\frac{dA}{dx} = 2x - \frac{432000}{x^2}$	$-$	0	$+$
grad	\	—	/

So $x = 60$ minimises the surface area.

Never try and evaluate anything with negative or fractional indices, put it back into positive index and/or root form first.

To find the n^th root of 216000 using your calculator, press:
216000 ^ (1 ÷ n) in tnis case n = 3

You cannot assume that your SP is the maximum or minimum, you must show this using the table of signs.