## Banker Question: No. 4

A function $f$ is defined by $f(x) = (2x-1)^5$

Find the coordinates of the stationary point on the graph with equation $y = f(x)$
and determine its nature. **( 7 )**

[Scroll to next page to see solution]

## Banker Question: No. 4

## Solution.

$$f(x) = (2x-1)^5$$

Differentiate:

$$f'(x) = 5(2x-1)^4 \times 2$$

$$f'(x) = 10(2x-1)^4$$

For an SP, $f'(x) = 0$ so

$$10(2x-1)^4 = 0$$

This gives $2x - 1 = 0 \quad \Rightarrow \quad x = \frac{1}{2}$

When $x = \frac{1}{2}$,

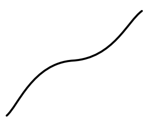
$$f(x) = \left(2\left(\tfrac{1}{2}\right) - 1\right)^5 \quad \Rightarrow \quad f(x) = (1-1)^5 = 0,$$

So $y = 0$. So SP is at $\left(\frac{1}{2},\ 0\right)$

Now use table of signs:

| | $x = 0$ | | $x = 1$ |
|---|---|---|---|
| $x$ | $\rightarrow$ | $\frac{1}{2}$ | $\rightarrow$ |
| $f'(x)$ | + | 0 | + |
| | / | – | / |

This is a point of inflexion.



Hence there is only one stationary point at $\left(\frac{1}{2},\ 0\right)$ which is a point of inflexion.

### Notes on solution

By looking at the question you should know immediately it is about differentiation and stationary points.

You should then be thinking:

For an SP $f'(x) = 0$

Table of signs.

Maximum, Minimum, Point of inflexion.

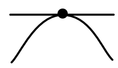
You should also recognise that this is a chain rule:

$$y = (\ )^n \quad \Rightarrow \quad \frac{dy}{dx} = n(\ )^{n-1} \times \frac{d(\ )}{dx}$$

$$y = (ax+b)^n \quad \Rightarrow \quad \frac{dy}{dx} = n(ax+b)^{n-1} \times a$$

For an SP $f'(x) = 0$.

In other words, the gradient is zero.



Here we have a 4th order polynomial.
( a polyonial of degree 4 – a quartic).
However it is already factorised because:

$$10(2x-1)^4 = 0$$

$$\rightarrow 10(2x-1)(2x-1)(2x-1)(2x-1) = 0$$

You are asked for the coordinates, so you need to process the y value by substituting back into the original equation for *f(x)*.

Remember *f(x)* is the function notation and *y* = .... is the graph notation.

They are both equivalent.

Use the table of signs to look at the gradient on either side of the SP.

Evaluate the derivative on either side to see if it is positive or negative (usually pick a convenient point on either side). The two points shown allow easy calculation.

$$f'(x) = 10(2x-1)^4$$

When $x = 0$ $\quad f'(x) = 10(0-1)^4 = 10 = +$

When $x = 1$ $\quad f'(x) = 10(2-1)^4 = 10 = +$

So SP is a point of inflexion, it is not a turning point.