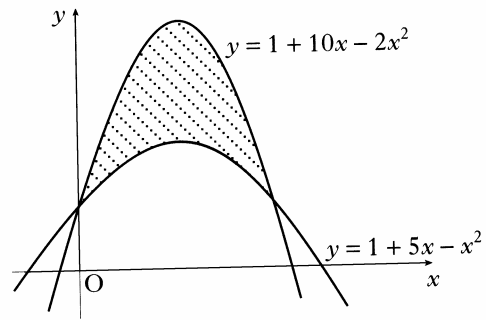


Banker Question: No. 6

Calculate the shaded area between the parabolas
with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$

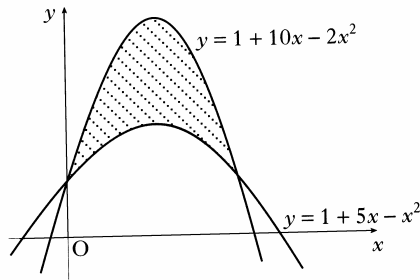
(6)



[Scroll to next page to see solution]

Banker Question: No. 6

Solution.



Find x -coordinates of points of intersection:

$$1 + 10x - 2x^2 = 1 + 5x - x^2$$

bring all terms to left hand side.

$$1 + 10x - 2x^2 - 1 - 5x + x^2 = 0$$

Simplify

$$5x - x^2 = 0$$

Factorise

$$x(5 - x) = 0 \quad \Rightarrow \quad x = 0 \quad \text{or} \quad x = 5$$

Form the integral

$$\int_0^5 (1 + 10x - 2x^2) - (1 + 5x - x^2) \, dx$$

Simplify

$$\int_0^5 1 + 10x - 2x^2 - 1 - 5x + x^2 \, dx$$

$$\rightarrow \int_0^5 5x - x^2 \, dx$$

Integrate

$$\rightarrow \left[\frac{5x^2}{2} - \frac{x^3}{3} \right]_0^5$$

Evaluate

$$\rightarrow \left(\frac{5(5)^2}{2} - \frac{(5)^3}{3} \right) - \left(\frac{5(0)^2}{2} - \frac{0^3}{3} \right)$$

$$\rightarrow \left(\frac{125}{2} - \frac{125}{3} \right) - 0$$

$$\rightarrow \frac{375}{6} - \frac{250}{6} \quad \rightarrow \quad \frac{125}{6}$$

$$\rightarrow 20\frac{5}{6} \text{ unit}^2$$

Notes on solution

By looking at the question you should know immediately it is about integration and the area between two curves.

You should be thinking:

- Definite Integral
- Points of intersection
- Top curve – bottom curve
- Evaluate

We can see from the graph that one of the points of intersection is where $x = 0$.

However, we need to calculate the other x -coordinate.

To do this we solve the equations simultaneously, in this case simply put one equal to the other because they are both $y = \dots$

Form the integral.

Make sure you use

Top – Bottom

Use brackets because the $(-)$ sign will change all the signs in the second equation.

Do not forget $\dots dx$

**Simplify before you integrate.
It makes it easier.**

Don't forget to drop the integral sign once you have done the integration.

Put in the square brackets and the limits.

Use brackets when evaluating and keep everything in fraction form. Do not change to decimal.

Show all your working and keep on simplifying. Follow through. You will make less mistakes this way.

State your answer clearly.