Banker Question: No. 7

a) Express $3\cos(x°)+5\sin(x°)$ in the form $k\cos\left(x°-a°\right)$ where $k>0$ and $0\le a\le 90$ **(4)**

b) Hence solve the equation $3\cos(x°)+5\sin(x°)=4$ for $0\le x\le 90$ **(3)**

[Scroll to next page to see solution]

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Solution.

a) Express $3\cos(x°)+5\sin(x°)$ in the form $k\cos(x°-a°)$ where $k>0$ and $0\le a\le 90$

Expand

$$k\cos(x°-a°)=k\cos x\cos a+k\sin x\sin a$$

Compare coefficients:

$$k\cos a=3 \qquad k\sin a=5$$

Square and add:

$$k^2=3^2+5^2 \rightarrow k^2=34 \rightarrow k=\sqrt{34}$$

Divide:

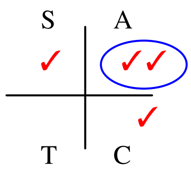
$$\tan a=\frac{5}{3} \quad \text{hence acute } a=\tan^{-1}\left(\frac{5}{3}\right)=59.03.....°$$

Determine the quadrant:

sine is + (so quadrants 1 & 2)
cosine is + (so quadrants 1 & 4)

Both are positive in quadrant 1.

So, $a=59°$



Hence: $3\cos(x°)+5\sin(x°)=\sqrt{34}\cos(x-59°)$

b) Hence solve the equation

$3\cos(x°)+5\sin(x°)=4$ for $0\le x\le 90$

From part *a)* $3\cos(x°)+5\sin(x°)=\sqrt{34}\cos(x-59°)$

So: $\sqrt{34}\cos(x-59°)=4$

$$\rightarrow \cos(x-59°)=\frac{4}{\sqrt{34}}$$

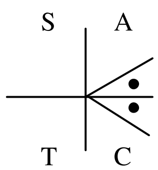
Hence acute $(x-59°)=\cos^{-1}\left(\frac{4}{\sqrt{34}}\right)=46.686...°$

Determine quadrants:

Cosine is +, so (1st & 4th quadrants)

$x - 59 = 46.7°$
$x - 59 = 360 - 46.7°$

So $x = 46.7 + 59 =$ **105.7**°
$x = 360 - 46.7 + 59 =$ **372.3**°



Neither of these are in the domain: $0\le x\le 90$

Subtract 360° and we obtain $372.3 - 360 = 12.3°$

The only solution in $0\le x\le 90$ is: $x = 12.3°$

Notes on solution

By looking at the question you should know immediately it is about trigonometry and the wave equation

You should recognise:

$$k\cos(x°-a°)$$

Look on formulae sheet ! Inside front cover.

To compare coefficients you can write:

$$k\cancel{\cos x}\cos a=3\cancel{\cos x} \rightarrow k\cos a=3$$
$$k\cancel{\sin x}\sin a=5\cancel{\sin x} \rightarrow k\sin a=5$$

Squaring and adding:

$$k^2\cos^2 a+k^2\sin^2 a=3^2+5^2$$
$$k^2(\cos^2 a+\sin^2 a)=3^2+5^2$$

$\cos^2 a+\sin^2 a=1$ This is always the case

Dividing: $\frac{k\sin a}{k\cos a}=\tan a$ This is always the case.

To determine which quadrant, you have to use:

$$k\cos a=3 \qquad k\sin a=5$$

You want the quadrant where both of these equations are true. Since sine and cosine are both positive, this only occurs in 1st quadrant.

Re-arrange the equation to solution form:

cos (.....) = a number.

DO NOT BREAK UP THE ARGUMENT (x – 59) until you have the 2 solutions.

Now you can separate the argument (x – 59).

The domain required is: $0\le x\le 90$

Since neither solution is in this domain, you can add or subtract the period of the wave – in this case 360° to see if you can find a solution within the domain. This is because both the sine and cosine wave are **periodic** functions.