Banker Question: No. 9

Before a forest fire was brought under control, the spread of fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double,
find the value of the constant k. **(3)**

[Scroll to next page to see solution]

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Solution.

Given equation $A = A_0 e^{kt}$

A_0 was the initial area of the fire.
A is the area covered t hours later.

It take 1½ hours for the area of the fire to double.

So when $t = 1.5$, the area was double the initial area
i.e. Area is now $2 \times A_0$

Put this into the equation.:

$$2A_0 = A_0 e^{k\times 1.5}$$

We can now cancel A_0

$$2\cancel{A_0} = \cancel{A_0} e^{k\times 1.5}$$

So

$$2 = e^{1.5k}$$

Take $\log_e$ of both sides

$$\log_e 2 = \log_e e^{1.5k}$$

Use rules of logs

$$\log_e 2 = 1.5k \log_e e$$

But

$$\log_e e = 1$$

So

$$\log_e 2 = 1.5k$$

Hence

$$\frac{\log_e 2}{1.5} = k$$

$$k = 0.462098...$$

$$k = 0.46 \text{ (2 d.p.)}$$

Notes on solution

By looking at the question you should know immediately it is about solving an exponential equation.

You should recall that to get to the variable in the index, you need to take logs of both sides.

Since the base is e, the obvious choice is $\log_e$ (or ln on your calculator).

The key to this question as in so many of this type is to use the relationship between value **NOW** (or the time you are interested in) and the **INITIAL** value.

i.e. Original area = A_0

Area now is double so it is $2A_0$

and you can cancel A_0 from both sides, thus removing it from the equation.

Divide by any constant multiplying the base, so that it is on its own before you take logs of each side. (Not required in this example)

Although not absolutely necessary, it saves considerable time in the calculations.

To get at the index, use the 3rd rule of logs:

$$\log_a x^p = p \log_a x$$

Remember also that $\log_e e = 1$

You are now left with a simple equation to solve.

Round the answer appropriately.