

Fractions and Equations

Reminder of fractions

We have used fractions with numbers before:

Add or subtract:

Change to a common denominator

e.g. $\frac{2}{3} + \frac{3}{4}$ change both fractions to a common denominator of 12

$$\rightarrow \frac{2}{3} \times \frac{4}{4} + \frac{3}{4} \times \frac{3}{3} \rightarrow \frac{8}{12} + \frac{9}{12} \rightarrow \frac{17}{12} \rightarrow 1\frac{5}{12}$$

Multiply:

Cancel down, **if you can**, then multiply tops (*numerators*), multiply bottoms (*denominators*).

e.g. $\frac{4}{9} \times \frac{3}{8}$ cancel down $\rightarrow \frac{\cancel{4}^1}{\cancel{9}^3} \times \frac{\cancel{3}^1}{\cancel{8}^2}$ multiply $\rightarrow \frac{1}{6}$

Divide:

Invert (*turn upside down*) the second fraction, change sign to multiply.

e.g. $\frac{5}{7} \div \frac{10}{21}$ invert 2nd fraction and change sign to multiply $\rightarrow \frac{5}{7} \times \frac{21}{10}$

Now treat as multiplication. Cancel down. $\rightarrow \frac{\cancel{5}^1}{\cancel{7}^1} \times \frac{\cancel{21}^3}{\cancel{10}^2}$

multiply: $\rightarrow \frac{3}{2} \rightarrow 1\frac{1}{2}$

Mixed numbers:

If adding or subtracting, deal with whole number parts separately.

e.g. $3\frac{2}{5} - 1\frac{2}{3}$ whole number part first: $\rightarrow 2 + \frac{2}{5} - \frac{2}{3}$

then as previously, using common denominator of 15 $\rightarrow 2 + \frac{6}{15} - \frac{10}{15}$

now deal with fraction: $\rightarrow 2 - \frac{4}{15} \rightarrow 1\frac{11}{15}$

Mixed numbers:

If multiplying or dividing,

YOU MUST CHANGE THE MIXED NUMBERS TO IMPROPER (Top Heavy) FRACTIONS.

e.g. $3\frac{2}{5} \times 1\frac{2}{3}$ change to improper fractions: $\rightarrow \frac{17}{5} \times \frac{5}{3}$

Cancel down, where you can. $\rightarrow \frac{17}{\cancel{5}^1} \times \frac{\cancel{5}^1}{3}$ and multiply $\rightarrow \frac{17}{3} \rightarrow 5\frac{2}{3}$

e.g. $2\frac{1}{2} \div 1\frac{1}{3}$ change to improper fractions: $\rightarrow \frac{5}{2} \div \frac{4}{3}$

invert and change to multiply: $\rightarrow \frac{5}{2} \times \frac{3}{4}$ multiply $\rightarrow \frac{15}{8} \rightarrow 1\frac{7}{8}$

Changing from a fraction to a mixed number:

To change an improper (*top heavy*) fraction to a mixed number:

Divide the top (*numerator*) by the bottom (*denominator*)

e.g. $\frac{9}{5} \rightarrow 1\frac{4}{5}$ because: $\frac{9}{5} \rightarrow \frac{5}{5} + \frac{4}{5} \rightarrow 1 + \frac{4}{5}$

e.g. $\frac{17}{3} \rightarrow 5\frac{2}{3}$ because: $\frac{17}{3} \rightarrow \frac{15}{3} + \frac{2}{3} \rightarrow 5 + \frac{2}{3}$

Changing from a mixed number to a fraction:

To change a mixed number to an improper (*top heavy*) fraction:

Multiply the whole number part by the denominator and add on the numerator.

e.g. $2\frac{2}{3} \rightarrow \frac{8}{3}$ because: $2\frac{2}{3} \rightarrow 2 + \frac{2}{3} \rightarrow \frac{6}{3} + \frac{2}{3} \rightarrow \frac{8}{3}$

e.g. $4\frac{1}{8} \rightarrow \frac{33}{8}$ because: $4\frac{1}{8} \rightarrow 4 + \frac{1}{8} \rightarrow \frac{32}{8} + \frac{1}{8} \rightarrow \frac{33}{8}$

e.g. $3\frac{3}{4} \rightarrow$ $\begin{array}{c} = 12 \quad \xrightarrow{+3} \quad = 15 \\ 3 \quad \frac{3}{4} \\ \quad \times \end{array} \rightarrow \frac{15}{4}$

Algebraic Fractions:

Simplifying fractions:

Divide the numerator and denominator by any common factor.

e.g. $\frac{8}{10} \rightarrow \frac{\cancel{8}^4}{\cancel{10}^5} \rightarrow \frac{4}{5}$ so, $\frac{2n}{3n} \rightarrow \frac{\cancel{2}^1}{\cancel{3}^1} \rightarrow \frac{2}{3}$

More examples:

$$\frac{x^2}{3x} \rightarrow \frac{\cancel{x}^2}{3x} \quad \text{divide by common factor } x \rightarrow \frac{x}{3}$$

$$\frac{2y}{y^2} \rightarrow \frac{\cancel{2}^1}{\cancel{y}^2} \quad \text{divide by common factor } y \rightarrow \frac{2}{y}$$

$$\frac{6xy}{2x} \rightarrow \frac{\cancel{6}^3 \cancel{x}^1 y}{\cancel{2}^1 \cancel{x}^1} \rightarrow \frac{3y}{1} \rightarrow 3y$$

$$\frac{5ab}{2ab^2} \rightarrow \frac{\cancel{5}^1 \cancel{a}^1 b}{\cancel{2}^1 \cancel{a}^1 b^2} \quad \text{divide by common factor } ab \rightarrow \frac{5}{2b}$$

Taking out a common factor first:

$$\frac{2m+6}{4} \quad \text{take out common factor} \rightarrow \frac{2(m+3)}{4} \rightarrow \frac{\cancel{2}^1 (m+3)}{\cancel{4}^2} \rightarrow \frac{m+3}{2}$$

$$\frac{2}{2x+4} \quad \text{take out common factor} \rightarrow \frac{2}{2(x+2)} \rightarrow \frac{\cancel{2}^1}{\cancel{2}^1 (x+2)} \rightarrow \frac{1}{x+2}$$

More involved factors

$$\frac{(x+y)(x-y)}{(x-y)} \quad \text{divide by common factor } (x-y) \rightarrow \frac{(x+y) \cancel{(x-y)}}{\cancel{(x-y)}} \rightarrow x+y$$

$$\frac{(t-1)(t+2)}{(t+2)(t-3)} \quad \text{divide by common factor } (t+2) \rightarrow \frac{(t-1) \cancel{(t+2)}}{\cancel{(t+2)}(t-3)} \rightarrow \frac{t-1}{t-3}$$

Further factorisation:

$$\frac{2a+8}{a+4} \quad \text{factorise top} \quad \rightarrow \quad \frac{2(a+4)}{a+4} \quad \text{cancel} \quad \rightarrow \quad \frac{2\cancel{(a+4)}}{\cancel{a+4}} \quad \rightarrow \quad 2$$

$$\frac{d^2+d}{d+1} \quad \text{factorise top} \quad \rightarrow \quad \frac{d(d+1)}{d+1} \quad \text{cancel} \quad \rightarrow \quad \frac{d\cancel{(d+1)}}{\cancel{d+1}} \quad \rightarrow \quad d$$

$$\frac{2t+2u}{3t+3u} \quad \text{factorise} \quad \rightarrow \quad \frac{2(t+u)}{3(t+u)} \quad \text{cancel} \quad \rightarrow \quad \frac{2\cancel{(t+u)}}{3\cancel{(t+u)}} \quad \rightarrow \quad \frac{2}{3}$$

$$\frac{m^2-mn}{5m-5n} \quad \text{factorise} \quad \rightarrow \quad \frac{m(m-n)}{5(m-n)} \quad \text{cancel} \quad \rightarrow \quad \frac{m\cancel{(m-n)}}{5\cancel{(m-n)}} \quad \rightarrow \quad \frac{m}{5}$$

Using difference of two squares:

$$\frac{x^2-1}{x-1} \quad \text{factorise} \quad \rightarrow \quad \frac{(x+1)(x-1)}{x-1} \quad \text{cancel} \quad \rightarrow \quad \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} \quad \rightarrow \quad x+1$$

$$\frac{m^2-9}{m+3} \quad \text{factorise} \quad \rightarrow \quad \frac{(m+3)(m-3)}{m+3} \quad \text{cancel} \quad \rightarrow \quad \frac{\cancel{(m+3)}(m-3)}{\cancel{m+3}} \quad \rightarrow \quad m-3$$

Try these: *(cover up the answers on the right first)*

$$\frac{y+4}{4y+16} \quad \rightarrow \quad \frac{y+4}{4(y+4)} \quad \rightarrow \quad \frac{1}{4}$$

$$\frac{2b+10}{b^2-25} \quad \rightarrow \quad \frac{2(b+5)}{(b+5)(b-5)} \quad \rightarrow \quad \frac{2}{b-5}$$

$$\frac{d^2+d}{d^3+d^2} \quad \rightarrow \quad \frac{d(d+1)}{d^2(d+1)} \quad \rightarrow \quad \frac{d}{d^2} \quad \rightarrow \quad \frac{1}{d}$$

$$\frac{4x+4x^2}{4x} \quad \rightarrow \quad \frac{4x(1+x)}{4x} \quad \rightarrow \quad 1+x$$

Algebraic Fractions:

Multiplication:

The same rules apply as with numbers:

Cancel down, **if you can**, then multiply tops (*numerators*), multiply bottoms (*denominators*).

e.g.	$\frac{m}{p} \times \frac{n}{r}$	nothing will cancel so multiply	$\rightarrow \frac{mn}{pr}$
e.g.	$\frac{u}{v} \times \frac{t}{u}$	cancel then multiply	$\rightarrow \frac{\cancel{u}}{v} \times \frac{t}{\cancel{u}} \rightarrow \frac{t}{v}$
e.g.	$\frac{k}{2} \times \frac{2}{n}$	cancel then multiply	$\rightarrow \frac{k}{\cancel{2}} \times \frac{\cancel{2}}{n} \rightarrow \frac{k}{n}$
e.g.	$\frac{a^2}{b} \times \frac{b}{a}$	cancel then multiply	$\rightarrow \frac{a^{\cancel{2}^1}}{b} \times \frac{\cancel{b}}{\cancel{a}} \rightarrow a$
e.g.	$\frac{u^3}{4} \times \frac{4}{u}$	cancel then multiply	$\rightarrow \frac{u^{\cancel{3}^2}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{u}} \rightarrow u^2$

Division:

The same rules apply as with numbers:

Invert (*turn upside down*) the second fraction, change sign to multiply.

e.g.	$\frac{a}{4} \div \frac{a}{3}$	invert, cancel, multiply	$\rightarrow \frac{a}{4} \times \frac{3}{a} \rightarrow \frac{\cancel{a}}{4} \times \frac{3}{\cancel{a}} \rightarrow \frac{3}{4}$
e.g.	$\frac{b^2}{2} \div \frac{b}{2}$	invert, cancel, multiply	$\rightarrow \frac{b^2}{2} \times \frac{2}{b} \rightarrow \frac{b^{\cancel{2}^1}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{b}} \rightarrow b$
e.g.	$\frac{u}{v} \div \frac{u}{v}$	invert, cancel, multiply	$\rightarrow \frac{u}{v} \times \frac{v}{u} \rightarrow \frac{\cancel{u}}{v} \times \frac{\cancel{v}}{\cancel{u}} \rightarrow 1$
e.g.	$\frac{3x}{2y} \div \frac{3}{4}$	invert, cancel, multiply	$\rightarrow \frac{3x}{2y} \times \frac{4}{3} \rightarrow \frac{\cancel{3}x}{\cancel{2}y} \times \frac{\cancel{4}^2}{\cancel{3}} \rightarrow \frac{2x}{y}$
e.g.	$\frac{2a^2}{3b^2} \div \frac{a^2}{b^2}$	invert, cancel, multiply	$\rightarrow \frac{2a^2}{3b^2} \times \frac{b^2}{a^2} \rightarrow \frac{2\cancel{a}^{\cancel{2}}}{3\cancel{b}^{\cancel{2}}} \times \frac{\cancel{b}^{\cancel{2}}}{\cancel{a}^{\cancel{2}}} \rightarrow \frac{2}{3}$
e.g.	$\frac{1}{u^3} \div \frac{1}{u^2}$	invert, cancel, multiply	$\rightarrow \frac{1}{u^3} \times \frac{u^2}{1} \rightarrow \frac{1}{u^{\cancel{3}^1}} \times \frac{\cancel{u}^2}{1} \rightarrow \frac{1}{u}$

Try these: (cover up the answers on the right first)

$$\frac{z}{4} \times \frac{z}{4} \rightarrow \frac{z^2}{16}$$

$$\frac{a}{b} \times \frac{a}{b} \rightarrow \frac{a^2}{b^2}$$

$$\frac{3m}{5} \times \frac{2n}{3m} \rightarrow \frac{2n}{5}$$

$$\frac{p^2}{3} \times \frac{2r}{p^3} \rightarrow \frac{2r}{3p}$$

$$\frac{3a^2}{2} \times \frac{4b}{a^3} \rightarrow \frac{6b}{a}$$

$$\frac{n}{6} \div \frac{n}{3} \rightarrow \frac{n}{6} \times \frac{3}{n} \rightarrow \frac{1}{2}$$

$$\frac{x^2}{4} \div \frac{x}{2} \rightarrow \frac{x^2}{4} \times \frac{2}{x} \rightarrow \frac{x}{2}$$

$$\frac{1}{t^2} \div \frac{2}{t} \rightarrow \frac{1}{t^2} \times \frac{t}{2} \rightarrow \frac{1}{2t}$$

$$\frac{x^2}{y} \div \frac{x}{y} \rightarrow \frac{x^2}{y} \times \frac{y}{x} \rightarrow x$$

$$\frac{m^2}{n} \times \frac{n}{m} \rightarrow m$$

$$\frac{2p^2}{q} \div \frac{3p}{2q^2} \rightarrow \frac{2p^2}{q} \times \frac{2q^2}{3p} \rightarrow \frac{4pq}{3}$$

$$14 \times \frac{t}{7} \rightarrow \frac{14}{1} \times \frac{t}{7} \rightarrow 2t$$

$$\frac{3u}{2v} \div \frac{v}{u} \rightarrow \frac{3u}{2v} \times \frac{u}{v} \rightarrow \frac{3u^2}{2v^2}$$

Algebraic Fractions:

Addition and subtraction:

The same rules apply as with numbers:

Change to a common denominator

e.g. $\frac{x}{5} + \frac{y}{2}$ use common denominator of 10 $\rightarrow \frac{x}{5} \times \frac{2}{2} + \frac{y}{2} \times \frac{5}{5} \rightarrow \frac{2x}{10} + \frac{5y}{10} \rightarrow \frac{2x+5y}{10}$

e.g. $\frac{u}{2} - \frac{3v}{4}$ use common denominator of 4 $\rightarrow \frac{u}{2} \times \frac{2}{2} - \frac{3v}{4} \rightarrow \frac{2u}{4} - \frac{3v}{4} \rightarrow \frac{2u-3v}{4}$

e.g. $\frac{5}{x} + \frac{2}{y}$ use common denominator of xy $\rightarrow \frac{5}{x} \times \frac{y}{y} + \frac{2}{y} \times \frac{x}{x} \rightarrow \frac{5y}{xy} + \frac{2x}{xy} \rightarrow \frac{5y+2x}{xy}$

e.g. $\frac{1}{m} - \frac{1}{n}$ use common denominator of mn $\rightarrow \frac{1}{m} \times \frac{n}{n} - \frac{1}{n} \times \frac{m}{m} \rightarrow \frac{n}{mn} - \frac{m}{mn} \rightarrow \frac{n-m}{mn}$

e.g. $\frac{x+1}{2} + \frac{x-1}{3}$ use common denominator of 6
 $\rightarrow \frac{x+1}{2} \times \frac{3}{3} + \frac{x-1}{3} \times \frac{2}{2} \rightarrow \frac{3(x+1)}{6} + \frac{2(x-1)}{6} \rightarrow \frac{3x+3+2x-2}{6} \rightarrow \frac{5x+1}{6}$

e.g. $\frac{1}{3x} + \frac{1}{4x}$ use common denominator of $12x$ $\rightarrow \frac{1}{3x} \times \frac{4}{4} + \frac{1}{4x} \times \frac{3}{3} \rightarrow \frac{4}{12x} + \frac{3}{12x} \rightarrow \frac{7}{12x}$

e.g. $\frac{4}{x} + \frac{3-x}{x^2}$ use common denominator of x^2 $\rightarrow \frac{4}{x} \times \frac{x}{x} + \frac{3-x}{x^2} \rightarrow \frac{4x}{x^2} + \frac{3-x}{x^2} \rightarrow \frac{3x+3}{x^2}$

e.g. $\frac{7}{x} - \frac{3}{x-3}$ use common denominator of $x(x-3)$
 $\rightarrow \frac{7}{x} \times \frac{x-3}{x-3} - \frac{3}{x-3} \times \frac{x}{x} \rightarrow \frac{7(x-3)}{x(x-3)} - \frac{3x}{x(x-3)} \rightarrow \frac{4x-21}{x(x-3)}$

e.g. $\frac{5}{p} + \frac{2}{p+2}$ use common denominator of $p(p+2)$
 $\rightarrow \frac{5}{p} \times \frac{p+2}{p+2} + \frac{2}{p+2} \times \frac{p}{p} \rightarrow \frac{5(p+2)}{p(p+2)} + \frac{2p}{p(p+2)} \rightarrow \frac{7p+10}{p(p+2)}$

Equations with Fractions:

A method:

Remove fractions

Remove brackets

Use these rules:

add or subtract the same number on each side

multiply or divide each side by the same number.

e.g. $\frac{x}{3} = \frac{1}{2}$ multiply both sides by 6 (l.c.m. of 3 and 2) $\frac{x}{3} \times 6 = \frac{1}{2} \times 6$

$\rightarrow 2x = 3$ which we can solve: $\rightarrow x = \frac{3}{2} \rightarrow x = 1\frac{1}{2}$

e.g. $y = \frac{2}{5}(y-1)$ multiply both sides by 5 $5 \times y = 5 \times \frac{2}{5}(y-1)$

$\rightarrow 5y = 2(y-1)$ which we can solve: $\rightarrow 5y = 2y - 2 \rightarrow 3y = -2 \rightarrow y = -\frac{2}{3}$

e.g. $\frac{x}{3} - \frac{x}{5} = 2$ multiply both sides by 15 $15 \times \frac{x}{3} - 15 \times \frac{x}{5} = 15 \times 2$

$\rightarrow 5x - 3x = 30$ which we can solve: $\rightarrow 2x = 30 \rightarrow x = 15$

e.g. $\frac{5x-3}{2} - \frac{x}{3} = 5$ multiply both sides by 6 $\rightarrow 6 \times \frac{5x-3}{2} - 6 \times \frac{x}{3} = 6 \times 5$

$\rightarrow 3(5x-3) - 2x = 30 \rightarrow 15x - 9 - 2x = 30 \rightarrow 13x = 39 \rightarrow x = 3$

e.g. $\frac{y+2}{2} + \frac{y+1}{4} = 2$ multiply both sides by 4 $\rightarrow 4 \times \frac{y+2}{2} + 4 \times \frac{y+1}{4} = 4 \times 2$

$\rightarrow 2(y+2) + (y+1) = 8 \rightarrow 2y + 4 + y + 1 = 8 \rightarrow 3y + 5 = 8 \rightarrow 3y = 3 \rightarrow y = 1$

e.g. $3x - \frac{(5x+4)}{3} = 4$ multiply both sides by 3 $3 \times 3x - 3 \times \frac{(5x+4)}{3} = 3 \times 4$

$9x - (5x+4) = 12 \rightarrow 9x - 5x - 4 = 12 \rightarrow 4x - 4 = 12 \rightarrow 4x = 16 \rightarrow x = 4$

Some Past Paper Questions:

Algebraic Fractions

1. Express as a single fraction in its simplest form $\frac{1}{2x} - \frac{1}{3x}, x \neq 0$
2. Express as a single fraction in its simplest form $\frac{3}{x} + \frac{2-x}{x^2}, x \neq 0$
3. Express as a single fraction in its simplest form $\frac{5}{x} - \frac{3}{(x-2)}, x \neq 0$ or $x \neq 2$

Solutions:

$$1. \quad \frac{1}{2x} - \frac{1}{3x} \rightarrow \frac{1}{2x} \times \frac{3}{3} - \frac{1}{3x} \times \frac{2}{2} \rightarrow \frac{3}{6x} - \frac{2}{6x} \rightarrow \frac{1}{6x}$$

$$2. \quad \frac{3}{x} + \frac{2-x}{x^2} \rightarrow \frac{3}{x} \times \frac{x}{x} + \frac{2-x}{x^2} \rightarrow \frac{3x}{x^2} + \frac{2-x}{x^2} \rightarrow \frac{3x+2-x}{x^2} \rightarrow \frac{2x+2}{x^2}$$

$$3. \quad \frac{5}{x} - \frac{3}{(x-2)} \rightarrow \frac{5}{x} \times \frac{x-2}{x-2} - \frac{3}{(x-2)} \times \frac{x}{x} \rightarrow \frac{5x-10}{x(x-2)} - \frac{3x}{x(x-2)} \rightarrow \frac{2x-10}{x(x-2)}$$

More past paper questions on next page

Some Past Paper Questions:

Fraction Equations

1. Solve the equation $\frac{2x+1}{3} - \frac{x}{4} = 2$
2. Solve the equation $\frac{x+4}{2} - \frac{2x+1}{3} = 1$, where x is a real number.
3. Solve **algebraically** the equation $3x - \frac{(5x+2)}{4} = 3$
4. Solve the equation $\frac{x-3}{2} + \frac{2x-1}{3} = 4$
5. Solve this equation for x : $\frac{x-2}{3} - \frac{x}{2} = \frac{1}{4}$
6. Solve **algebraically**, the equation $\frac{x}{2} - \frac{(x+1)}{3} = 4$
7. Solve **algebraically**, the equation $\frac{m}{3} = \frac{(1-m)}{5}$

Solutions:

1. $\frac{2x+1}{3} - \frac{x}{4} = 2$ *multiply throughout by 12* $\rightarrow 4(2x+1) - 3x = 24 \rightarrow x = 4$
2. $\frac{x+4}{2} - \frac{2x+1}{3} = 1$ *multiply throughout by 6* $\rightarrow 3(x+4) - 2(2x+1) = 6 \rightarrow x = 4$
3. $3x - \frac{5x+2}{4} = 3$ *multiply throughout by 4* $\rightarrow 6x - 5x - 2 = 12 \rightarrow x = 14$
4. $\frac{x-3}{2} + \frac{2x-1}{3} = 4$ *multiply throughout by 6* $\rightarrow 3(x-3) + 2(2x-1) = 24 \rightarrow x = 5$
5. $\frac{x-2}{3} - \frac{x}{2} = \frac{1}{4}$ *multiply throughout by 12* $\rightarrow 4(x-2) - 6x = 3 \rightarrow x = -5\frac{1}{2}$
6. $\frac{x}{2} - \frac{x+1}{3} = 4$ *multiply throughout by 6* $\rightarrow 3x - 2(x+1) = 24 \rightarrow x = 26$
7. $\frac{m}{3} = \frac{1-m}{5}$ *multiply throughout by 15* $\rightarrow 5m = 3 - 3m \rightarrow m = \frac{3}{8}$