Revision Sheet

Differentiation - <u>DECREASE</u> the power

Rules for Differentiation	
$f(x)$	$f'(x)$
x^n	nx^{n-1}
ax^n	anx^{n-1}
x	1
k	0
$f(x)+g(x)$	$f'(x)+g'(x)$
$(....)^n$	$n(.....)^{n-1}\times\frac{d}{dx}(.....)$

Rules for Differentiation	
$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

The examples below all use f(x) and f′(x) notation, however, the results are the same using dy/dx notation.

Cover up the answers and try them.

function	notes	derivative
$f(x)=5$	derivative of constant is 0	$f'(x)=0$
$f(x)=x$	derivative of x is 1	$f'(x)=1$
$f(x)=3x$	derivative of ax is a	$f'(x)=3$
$f(x)=x^4$	a basic power of x	$f'(x)=4x^3$
$f(x)=7x^3$	power of x with multiplier	$f'(x)=7\times3x^2 = 21x^2$
$f(x)=\frac{1}{2}x^2$	power of x with multiplier	$f'(x)=2\times\frac{1}{2}x = x$
$f(x)=x^3-4x^2+5x-1$	sum of basic powers	$f'(x)=3x^2-8x+5$
$f(x)=(2x-1)(x+3)$	multiply brackets out $f(x)=2x^2+5x-3$	$f'(x)=4x+5$
$f(x)=(x+1)^2$	multiply brackets out $f(x)=x^2+2x+1$	$f'(x)=2x+2$
$f(x)=x^{-3}$	negative power of x	$f'(x)=-3x^{-4}$
$f(x)=\frac{1}{x}$	straight line form $f(x)=x^{-1}$	$f'(x)=-1\cdot x^{-2} \rightarrow -\frac{1}{x^2}$
$f(x)=\frac{1}{x^2}$	straight line form $f(x)=x^{-2}$	$f'(x)=-2x^{-3} \rightarrow -\frac{2}{x^3}$
$f(x)=x^{\frac{1}{2}}$	fractional indices	$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$
$f(x)=x^{-\frac{3}{2}}$	negative fractional index	$f'(x)=-\frac{3}{2}x^{-\frac{5}{2}}$
$f(x)=3x^{-4}$	multiplier with negative index	$f'(x)=-4\times3x^{-5} = -12x^{-5}$
$f(x)=2x^{\frac{2}{3}}$	$f'(x)=\frac{2}{3}\times2x^{-\frac{1}{3}} \rightarrow \frac{4}{3}x^{-\frac{1}{3}}$	$\rightarrow f'(x) = \frac{4}{3x^{\frac{1}{3}}} \rightarrow \frac{4}{3\sqrt[3]{x}}$

function	notes	derivative
$f(x)=5x^{-\frac{1}{3}}$	$f'(x)=-\frac{1}{3}\times 5x^{-\frac{4}{3}} \rightarrow \dfrac{5}{3x^{\frac{4}{3}}}$	$\rightarrow \quad f'(x)= -\dfrac{5}{3\left(\sqrt[3]{x}\right)^4}$
$f(x)=\sqrt{x}$	straight line form $f(x)=x^{\frac{1}{2}}$	$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$
$f(x)=\sqrt[3]{x^2}$	straight line form $f(x)=x^{\frac{2}{3}}$	$f'(x)=\frac{2}{3}x^{-\frac{1}{3}} \rightarrow \dfrac{2}{3\sqrt[3]{x}}$
$f(x)=\sqrt[4]{x^5}$	straight line form $f(x)=x^{\frac{5}{4}}$	$f'(x)=\frac{5}{4}x^{\frac{1}{4}} \rightarrow \frac{5}{4}\sqrt[4]{x}$
$f(x)=x^{\frac{1}{2}}\left(1-x^{\frac{1}{2}}\right)$	multiply out $f(x)=x^{\frac{1}{2}}-x^{\frac{1}{2}}\cdot x^{\frac{1}{2}} = x^{\frac{1}{2}}-x$	$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-1 \rightarrow \dfrac{1}{2\sqrt{x}}-1$
$f(x)=\dfrac{1}{\sqrt{x}}$	straight line form $f(x)=x^{-\frac{1}{2}}$	$f'(x)=-\frac{1}{2}x^{-\frac{3}{2}} \rightarrow -\dfrac{1}{2\left(\sqrt{x}\right)^3}$
$f(x)=\dfrac{3}{2\sqrt{x}}$	str. line form $f(x)=\dfrac{3}{2}\times\dfrac{1}{\sqrt{x}}=\frac{3}{2}x^{-\frac{1}{2}}$	$f'(x)=-\dfrac{1}{2}\times\dfrac{3}{2}x^{-\frac{3}{2}} \rightarrow -\dfrac{3}{4\left(\sqrt{x}\right)^3}$
$f(x)=\left(1+\sqrt{x}\right)^2$	multiply out $f(x)=1+2\sqrt{x}+x = 1+2x^{\frac{1}{2}}+x$	$f'(x)=\frac{1}{2}\cdot 2x^{-\frac{1}{2}}+1 \rightarrow \dfrac{1}{\sqrt{x}}+1$
$f(x)=\dfrac{3x^3-2x}{x^2}$	split fraction $f(x)=\dfrac{3x^3}{x^2}-\dfrac{2x}{x^2} = 3x-2x^{-1}$	$f'(x)=3-(-1)2x^{-2} \rightarrow 3+\dfrac{2}{x^2}$
$f(x)=(x+4)^3$	simple chain rule	$f'(x)=3(x+4)^2\times 1 \rightarrow 3(x+4)^2$
$f(x)=(2x-3)^4$	chain rule	$f'(x)=4(2x-3)^3\cdot 2 \rightarrow 8(2x-3)^3$
$f(x)=5(3x-1)^3$	chain rule	$f'(x)=5(3x-1)^2\cdot 3 \rightarrow 15(3x-1)^2$
$f(x)=(x^3+5)^4$	chain rule	$f'(x)=2(x^3+5)^3\cdot 3x^2 \rightarrow 6x^2(x^3+5)^3$
$f(x)=(3x^2-5)^5$	chain rule	$f'(x)=5(3x^2-5)^4\cdot 6x \rightarrow 30x(3x^2-5)^4$
$f(x)=4(2x^3-5)^3$	chain rule	$f'(x)=3\times 4(2x^3-5)^2\cdot 6x^2 \rightarrow 72x^2(2x^3-5)^2$
$f(x)=(3x-1)^{-2}$	chain rule	$f'(x)=-2(3x-1)^{-3} \rightarrow -\dfrac{2}{(3x-1)^3}$
$f(x)=(x^2+2)^{-1}$	chain rule	$f'(x)=-1(x^2+2)^{-2}\cdot 2x \rightarrow -\dfrac{2x}{(x^2+2)^2}$
$f(x)=(2x+1)^{\frac{1}{2}}$	chain rule	$f'(x)=\frac{1}{2}\left(2x+1\right)^{-\frac{1}{2}}\cdot 2 \rightarrow \dfrac{1}{\sqrt{2x+1}}$
$f(x)=(x^2+3)^{\frac{1}{3}}$	chain rule	$f'(x)=\dfrac{1}{3}(x^2+3)^{-\frac{2}{3}}\cdot 2x \rightarrow \dfrac{2x}{3\left(\sqrt[3]{x^2+3}\right)^2}$
$f(x)=(x^2+2x)^3$	chain rule	$f'(x)=3(x^2+2x)^2\cdot(2x+2) \rightarrow 3(2x+2)(x^2+2x)^2$
$f(x)=(x^2-x)^{-1}$	chain rule	$f'(x)=-1(x^2-x)^{-2}\cdot(2x-1) \rightarrow -\dfrac{2x-1}{\left(x^2-x\right)^2}$
$f(x)=\dfrac{1}{(2x-1)}$	S.L.F. C.R. $f(x)=(2x-1)^{-1}$	$f'(x)=-1(2x-1)^{-2}\cdot 2 \rightarrow -\dfrac{2}{(2x-1)^2}$
$f(x)=\dfrac{3}{4(2x+5)}$	S.L.F. C.R. $f(x)=\frac{3}{4}(2x+5)^{-1}$	$f'(x)=(-1)\cdot\frac{3}{4}(2x+5)^{-2}\cdot 2 \rightarrow -\dfrac{3}{2(2x+5)^2}$
$f(x)=\dfrac{1}{(3x+2)^3}$	S.L.F. C.R. $f(x)=(3x+2)^{-3}$	$f'(x)=-3(3x+2)^{-4}\cdot 3 \rightarrow -\dfrac{9}{(3x+2)^4}$

function	notes	derivative
$f(x)=\sqrt[3]{(1+6x)}$	$f(x)=(x+1)^{-\frac{1}{2}}$	$f'(x)=-\frac{1}{2}(x+1)^{-\frac{3}{2}} \rightarrow -\frac{1}{2\left(\sqrt{x+1}\right)^3}$
$f(x)=\frac{3}{\sqrt[3]{9-x}}$	$f(x)=3(9-x)^{-\frac{1}{3}}$	$f'(x)=-\frac{1}{3}\cdot 3(9-x)^{-\frac{4}{3}}\cdot(-1) \rightarrow \frac{1}{\left(\sqrt[3]{9-x}\right)^4}$
$f(x)=\left(1+\frac{1}{x}\right)^4$	$f(x)=\left(1+x^{-1}\right)^4$	$f'(x)=4\left(1+x^{-1}\right)^3\cdot(-x^{-2}) \rightarrow -\frac{4\left(1+\frac{1}{x}\right)^3}{x^2}$
$f(x)=\sqrt{2x^2-4x+6}$	$f(x)=\left(2x^2-4x+6\right)^{\frac{1}{2}}$	$f'(x)=\frac{1}{2}\left(2x^2-4x+6\right)^{-\frac{1}{2}}\cdot(4x-4) \rightarrow \frac{(4x-4)}{2\sqrt{2x^2-4x+6}}$
$f(x)=\frac{1}{\sqrt{x+1}}$	$f(x)=(x+1)^{-\frac{1}{2}}$	$f'(x)=-\frac{1}{2}(x+1)^{-\frac{3}{2}}\cdot 1 \rightarrow -\frac{1}{2\left(\sqrt{(x+1)}\right)^3}$
$f(x)=\frac{1}{2x}+\frac{1}{2x-1}$	$f(x)=\frac{1}{2}x^{-1}+\left(2x-1\right)^{-1}$	$f'(x)=-\frac{1}{2}x^{-2}+(-1)(2x-1)^{-2}\cdot 2 \rightarrow -\frac{1}{2x^2}-\frac{2}{(2x-1)^2}$
$f(x)=\frac{1}{\sqrt{x+1}}$	$f(x)=(x+1)^{-\frac{1}{2}}$	$f'(x)=-\frac{1}{2}\left(x+1\right)^{-\frac{3}{2}}\cdot 1 \rightarrow -\frac{1}{\left(\sqrt{x+1}\right)^3}$
$f(x)=3\sin x$	Basic form with multiplier	$f'(x)=3\cos x$
$f(x)=1-\sin x$	derivative of 1 is 0	$f'(x)=-\cos x$
$f(x)=2+\cos x$	derivative of 2 is 0	$f'(x)=-\sin x$
$f(x)=\frac{1}{2}\cos x$	Basic form with multiplier	$f'(x)=-\frac{1}{2}\sin x$
$f(x)=-2\cos x$	Basic form with multiplier	$f'(x)=2\sin x$
$f(x)=4\sin x+7\cos x$	f(x) + g(x) with multipliers	$f'(x)=4\cos x-7\sin x$
$f(x)=5x^2-\sin x$	f(x) + g(x) with multipliers	$f'(x)=10x-\cos x$
$f(x)=\sqrt{x}+\cos x$	f(x) + g(x)	$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-\sin x \rightarrow \frac{1}{2\sqrt{x}}-\sin x$
$f(x)=\frac{2+x\sin x}{x}$	$f(x)=\frac{2}{x}+\frac{x\sin x}{x}=2x^{-1}+\sin x$	$f'(x)=-2x^{-2}+\cos x \rightarrow -\frac{2}{x^2}-\sin x$
$f(x)=\frac{3+x^2\cos x}{x}$	$f(x)=\frac{3}{x^2}+\frac{x^2\cos x}{x^2}=3x^{-2}+\cos x$	$f'(x)=-6x^{-3}-\sin x \rightarrow -\frac{6}{x^3}-\cos x$
$f(x)=\sin 2x$	Chain rule	$f'(x)=2\cos 2x$
$f(x)=\cos 4x$	Chain rule	$f'(x)=-4\sin 4x$
$f(x)=\sin\frac{1}{2}x$	Chain rule	$f'(x)=\frac{1}{2}\cos\frac{1}{2}x$
$f(x)=\sin(2x-3)$	Chain rule	$f'(x)=\cos(2x-3)\cdot 2 \rightarrow 2\cos(2x-3)$
$f(x)=\cos(x^2-1)$	Chain rule	$f'(x)=-\sin(x^2-1)\cdot 2x \rightarrow -2x\sin(x^2-1)$
$f(x)=\frac{1}{\cos x}$	$f(x)=(\cos x)^{-1}$	$f'(x)=-1(\cos x)^{-2}(-\sin x) \rightarrow \frac{\sin x}{\cos^2 x}$

function	notes	derivative
$f(x)=\dfrac{3}{4\sin x}$	$f(x)=\dfrac{3}{4}(\sin x)^{-1}$	$f'(x)=(-1)\,\frac{3}{4}(\sin x)^{-2}\cos x \;\rightarrow\; \dfrac{-3\cos x}{4\sin^2 x}$
$f(x)=\sin^2 x$	$f(x)=(\sin x)^2$	$f'(x)=2\sin x\cos x \;\rightarrow\; \sin 2x$
$f(x)=\cos^2 x$	$f(x)=(\cos x)^2$	$f'(x)=-2\cos x\sin x \;\rightarrow\; -\sin 2x$
$f(x)=\sin^4 x$	$f(x)=(\sin x)^4$	$f'(x)=4(\sin x)^3\cos x \;\rightarrow 4\cos x\sin^3 x$
$f(x)=(1+\sin x)^2$	Chain rule	$f'(x)=2(1+\sin x)\cos x \;\rightarrow\; 2\cos x(1+\sin x)$
$f(x)=\sqrt{1+\sin x}$	$f(x)=(1+\sin x)^{\frac{1}{2}}$	$f'(x)=\frac{1}{2}(1+\sin x)^{-\frac{1}{2}}\cdot\cos x \;\rightarrow\; \dfrac{\cos x}{2\sqrt{1+\sin x}}$
$f(x)=2\sin x\cos x$	$f(x)=\sin 2x$	$f'(x)=2\cos 2x$

Extra Practice Examples from Maths in Action:

Basic differentiation	p. 62	Ex 2:	Qu. 1 - 11
Negative and fractional powers	p. 64	Ex 4A:	Qu. 1 - 6
Fractions and roots	p. 65	Ex 4B:	Qu. 1 - 8
Basic trig functions	p. 221	Ex 2A:	Qu. 1 - 4
More trig functions	p. 222	Ex 2B:	Qu. 1 - 4
Chain Rule	p. 225	Ex 3A:	Qu. 1 - 7
More Chain Rule	p. 226	Ex 3B:	Qu. 1 - 5
Chain Rules for Trig	p. 227	Ex 4A:	Qu. 1 - 13