Indices

Rules of Indices:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$\left(a^m\right)^n = a^{mn}$$

Examples of using the rules:

1. Multiplying – add the indices

   e.g. $x^4 \times x^5 \rightarrow x^9$

2. Dividing – subtract the indices

   e.g. $z^7 \div z^5 \rightarrow z^2$ and $\frac{p^6}{p^2} \rightarrow p^4$

3. Power of a power – multiply the indices

   e.g. $\left(x^3\right)^2 \rightarrow x^6$

Some useful results – Special indices

1.	$a^0 \rightarrow 1$	anything raised to the power 0 is 1	e.g.	$3^0 \rightarrow 1$	$x^0 \rightarrow 1$
2.	$a^1 \rightarrow a$	anything raised to the power 1 is itself	e.g.	$5^1 \rightarrow 5$	$x^1 \rightarrow x$
		note the reverse:	i.e.	$m \rightarrow m^1$	

Negative and fractional indices

1. $a^{-1} \rightarrow \frac{1}{a}$ and $a^{-2} \rightarrow \frac{1}{a^2}$ in general $a^{-n} \rightarrow \frac{1}{a^n}$ 'minus' - means '1 over'

2. $a^{\frac{1}{2}} \rightarrow \sqrt{a}$ and $a^{\frac{1}{3}} \rightarrow \sqrt[3]{a}$ in general $a^{\frac{1}{n}} \rightarrow \sqrt[n]{a}$

   the denominator in the fraction denotes which root it is.

   e.g. $\sqrt{16} = 4$ (since $4 \times 4 = 16$) and $\sqrt[3]{27} = 3$ (since $3 \times 3 \times 3 = 27$)

   $\sqrt[3]{8} = 2$ (since $2 \times 2 \times 2 = 8$) and $\sqrt[4]{16} = 2$ (since $2 \times 2 \times 2 \times 2 = 16$)

3. $a^{\frac{2}{3}} \rightarrow \sqrt[3]{a^2}$ *or* $\left(\sqrt[3]{a}\right)^2$ in general $a^{\frac{m}{n}} \rightarrow \sqrt[n]{a^m}$ *or* $\left(\sqrt[n]{a}\right)^m$

   the <u>denominator</u> gives the root, the <u>numerator</u> raises it to the power.

Some examples of applications

1. $(pq)^3$ this means $pq \times pq \times pq \rightarrow p \times q \times p \times q \times p \times q \rightarrow p^3q^3$

2. $(2a)^4$ this means $2a \times 2a \times 2a \times 2a \rightarrow 16a^4$

3. $2a^4$ this means $2 \times a \times a \times a \times a \rightarrow 2a^4$ do not confuse this with the one above.

4. $\frac{5}{(5p)^3}$ simplify the denominator, then cancel. $\frac{5}{5 \times 5 \times 5 \times p^3} \rightarrow \frac{\cancel{5}^1}{\cancel{5}^1 \times 5 \times 5 \times p^3} \rightarrow \frac{1}{25p^3}$

5. $(ab^3)^2$ this means $ab^3 \times ab^3 \rightarrow a \times b^3 \times a \times b^3 \rightarrow a^2b^6$

6. $3x^2 \times 5x^2$ deal with numbers first. $\rightarrow 3 \times 5 \times x^2 \times x^2 \rightarrow 15x^4$

7. $12y^3 \div 6y$ deal with numbers first. $12 \div 6 \rightarrow 2$ and $y^3 \div y \rightarrow y^2$ so $\rightarrow 2y^2$

   or in fraction form and cancel $\rightarrow \frac{12y^3}{6y} \rightarrow \frac{\cancel{12}^2\ \cancel{y^3}^{y^2}}{\cancel{6}^1\ \cancel{y}^1} \rightarrow 2y^2$

8. $w^3\left(w^4+w^3\right)$ Multiply out the brackets in the usual way. $\rightarrow w^3\times w^4+w^3\times w^3 \rightarrow w^7+w^6$

9. $\dfrac{u^4\times u^5}{u^3}$ Numerator first $\rightarrow \dfrac{u^9}{u^3} \rightarrow u^6$

10. $\dfrac{s^2\times s^{-4}}{s^{-3}}$ Numerator first $\rightarrow \dfrac{s^{-2}}{s^{-3}} \rightarrow s^{-2-(-3)} \rightarrow s^1 \rightarrow s$

11. $5^n=25$ What power do we raise 5 to get 25, n must be 2, so $n=2$
At the moment, take a good guess and check it out. They are usually easy numbers.

Alternatively, put the right hand side in index form and compare indices.

$5^n=5^2$ so clearly $n=2$

12. $4^n=\dfrac{1}{4}$ Put the right hand side in index form. $4^n=4^{-1}$ so clearly $n=-1$

13. $4x^{-5}$ Writing with positive indices, get rid of the '–1' by using '1 over'.

$\rightarrow 4\times\dfrac{1}{x^5} \rightarrow \dfrac{4}{x^5}$ Note the '–5' power only applies to the x

Treat the number as a separate multiplier.

14. $\dfrac{2}{3}v^{-4}$ Writing with positive indices, get rid of the '–1' by using '1 over'.

Treat the number as a separate multiplier.

$\rightarrow \dfrac{2}{3}\times v^{-4} \rightarrow \dfrac{2}{3}\times\dfrac{1}{v^4} \rightarrow \dfrac{2}{3v^4}$

15. $t^{\frac{3}{4}}$ Writing in root form $\rightarrow \sqrt[4]{t^3}$ *or* $\left(\sqrt[4]{t}\right)^3$

16. $m^{-\frac{4}{5}}$ Writing in root form $\rightarrow \dfrac{1}{m^{\frac{4}{5}}}$ (negative index first) $\rightarrow \dfrac{1}{\sqrt[5]{m^4}}$ *or* $\dfrac{1}{\left(\sqrt[5]{m}\right)^4}$

17. $\sqrt[3]{p^4}$ Writing in index form $\rightarrow$ $p^{\frac{4}{3}}$ (check it goes back to the root form)

Remember: denominator = root, numerator = power.

18. $16^{\frac{1}{4}}$ Evaluate. Put into root form. $\rightarrow$ $\sqrt[4]{16}$ $\rightarrow$ 2 then check it. ($2 \times 2 \times 2 \times 2 = 16$)

19. $27^{\frac{2}{3}}$ Evaluate. Put into root form. $\rightarrow$ $\left(\sqrt[3]{27}\right)^2$ $\rightarrow$ 3^2 $\rightarrow$ 9

$\left(\sqrt[3]{27}\right)^2$ is easier to work with than $\sqrt[3]{27^2}$.

20. $v^{\frac{5}{2}} \times v^{\frac{1}{2}}$ Simplify – use the rules. Add the indices. $v^{\frac{5}{2}+\frac{1}{2}}$ $\rightarrow$ $v^{\frac{6}{2}}$ $\rightarrow$ v^3

21. $x^{\frac{3}{5}} \div x^{-\frac{1}{5}}$ Simplify – use the rules. Subtract the indices. $x^{\frac{3}{5}-\left(-\frac{1}{5}\right)}$ $\rightarrow$ $x^{\frac{3}{5}+\frac{1}{5}}$ $\rightarrow$ $x^{\frac{4}{5}}$

22. $\left(p^{-\frac{1}{2}}\right)^3$ Simplify – use the rules. Multiply the indices. $\rightarrow$ $p^{-\frac{3}{2}}$

23. $\left(a^2+1\right)\left(a^{-2}+1\right)$ Use FOIL and the rules in the normal way.

$\rightarrow$ $a^2 \times a^{-2} + a^2 \times 1 + 1 \times a^{-2} + 1 \times 1$ $\rightarrow$ $a^0 + a^2 + a^{-2} + 1$

$\rightarrow$ $1 + a^2 + a^{-2} + 1$ $\rightarrow$ $2 + a^2 + a^{-2}$

24. $\left(x^{\frac{1}{2}}+1\right)\left(x^{\frac{1}{2}}+1\right)$ Use FOIL and the rules in the normal way.

$\rightarrow$ $x^{\frac{1}{2}} \times x^{\frac{1}{2}} + x^{\frac{1}{2}} \times 1 + 1 \times x^{\frac{1}{2}} + 1 \times 1$ $\rightarrow$ $x^{\frac{1}{2}+\frac{1}{2}} + x^{\frac{1}{2}} + x^{\frac{1}{2}} + 1$

$\rightarrow$ $x^1 + 2x^{\frac{1}{2}} + 1$ $\rightarrow$ $x + 2x^{\frac{1}{2}} + 1$