Surds

What is a surd?

A surd is an irrational number – a number that cannot be expressed as a fraction.

In simple terms – it is the square root of a number that cannot be evaluated as a whole number.

e.g. These are surds: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$ *etc*

These are NOT surds: $\sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}$, *etc* since these are 2, 3, 4, 5 respectively.

Rules of Surds:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

These can be used in both directions

Examples of using the rules:

1. **Multiplication rule**

e.g. $\sqrt{3} \times \sqrt{2} \rightarrow \sqrt{3 \times 2} \rightarrow \sqrt{6}$

in reverse we can also say: $\sqrt{20} \rightarrow \sqrt{4 \times 5} \rightarrow \sqrt{4} \times \sqrt{5} \rightarrow 2\sqrt{5}$

This is a useful way of simplifying surds.

2. **Division rule**

e.g. $\frac{\sqrt{24}}{\sqrt{3}} \rightarrow \sqrt{\frac{24}{3}} \rightarrow \sqrt{8}$ Note that we can further simplify this:

$\sqrt{8} \rightarrow \sqrt{4 \times 2} \rightarrow \sqrt{4} \times \sqrt{2} \rightarrow 2\sqrt{2}$

Also the other way: $\sqrt{\frac{72}{9}} \rightarrow \frac{\sqrt{72}}{\sqrt{9}} \rightarrow \frac{\sqrt{72}}{3}$

We can also further simplify this: $\frac{\sqrt{72}}{3} \rightarrow \frac{\sqrt{36 \times 2}}{3} \rightarrow \frac{\sqrt{36} \times \sqrt{2}}{3} \rightarrow \frac{6 \times \sqrt{2}}{3} \rightarrow 2\sqrt{2}$

Useful tip: $\sqrt{2} \times \sqrt{2} = 2$ and in general: $\sqrt{a} \times \sqrt{a} = \left(\sqrt{a}\right)^2 = \sqrt{a^2} = a$

Applications

Simplification:

Simplify: $\sqrt{54}$

To break this up, we look for the largest square number that is a factor.

e.g. 54 = 2 × 27 or 9 × 6 or 3 × 18

The quickest way is to see which of the **square numbers**

i.e. 4, 9, 16, 25, 36, 49, 64, etc. will go into it.

In this example 9 is the largest square factor.

$\sqrt{54} = \sqrt{9\times6}$ now use the rule to split it. $\sqrt{9\times6} \rightarrow \sqrt{9}\times\sqrt{6} \rightarrow 3\sqrt{6}$

Adding or subtracting surds

You can only add 'like' surds – just as in algebra you can only add or subtract 'like' terms.

$3\sqrt{2}+5\sqrt{2} \rightarrow 8\sqrt{2}$ and $5\sqrt{3}-2\sqrt{3} \rightarrow 3\sqrt{3}$ also: $3\sqrt{5}-\sqrt{5} \rightarrow 2\sqrt{5}$

Sometimes we must simplify before we can add or subtract.

$\sqrt{18}-\sqrt{2}$

Simplify the $\sqrt{18}$ term by looking for the largest square factor.

$\sqrt{18}-\sqrt{2} \rightarrow \sqrt{9\times2}-\sqrt{2} \rightarrow \sqrt{9}\times\sqrt{2}-\sqrt{2} \rightarrow 3\sqrt{2}-\sqrt{2} \rightarrow 2\sqrt{2}$

$\sqrt{63}-\sqrt{28}$

Simplify both terms by looking for the largest square factor.

$\sqrt{63}-\sqrt{28} \rightarrow \sqrt{9\times7}-\sqrt{4\times7} \rightarrow \sqrt{9}\sqrt{7}-\sqrt{4}\sqrt{7} \rightarrow 3\sqrt{7}-2\sqrt{7} \rightarrow \sqrt{7}$

Simplification

$\sqrt{3}\times\sqrt{15}$

Combine the two surds and look to simplify further.

$\sqrt{3}\times\sqrt{15} \rightarrow \sqrt{3\times15} \rightarrow \sqrt{45} \rightarrow \sqrt{9\times5} \rightarrow \sqrt{9}\sqrt{5} \rightarrow 3\sqrt{5}$

By combining the two surds – we can often find a square factor to simplify with.

Multiplying out brackets

$\left(2+\sqrt{2}\right)\left(3+\sqrt{2}\right)$ Just use FOIL and the rules of surds.

$$\left(2+\sqrt{2}\right)\left(3+\sqrt{2}\right) \rightarrow 2\times3 + 2\sqrt{2} + 3\sqrt{2} + \sqrt{2}\sqrt{2} \rightarrow 6 + 5\sqrt{2} + 2 \rightarrow 8+5\sqrt{2}$$

More involved brackets:

$$\left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}-\sqrt{2}\right) \rightarrow \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} - \sqrt{2}\frac{1}{\sqrt{2}} + \sqrt{2}\frac{1}{\sqrt{2}} - \sqrt{2}\sqrt{2} \rightarrow \frac{1}{2} - 1 + 1 - 2 \rightarrow 1\frac{1}{2}$$

Rationalising the denominator.

Surds are often in denominators and are sometimes inconvenient there.

We can remove them as follows:

e.g. $\frac{3}{\sqrt{2}}$ To rationalise this (i.e. get rid of the surd in the denominator)

Multiply top and bottom by the surd i.e. $\sqrt{2}$

$$\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{3\sqrt{2}}{\sqrt{2\times2}} \rightarrow \frac{3\sqrt{2}}{\sqrt{4}} \rightarrow \frac{3\sqrt{2}}{2}$$

This will work for any simple surd in the denominator.

However, we often have more involved surds such as $2+\sqrt{2}$.

In this case we multiply by the conjugate (conjugate means opposite sign)

So the conjugate of $2+\sqrt{2}$ is $2-\sqrt{2}$

e.g. $\frac{1}{\sqrt{7}-1}$ multiply top and bottom by the conjugate $\frac{1}{\sqrt{7}-1} \rightarrow \frac{1}{\sqrt{7}-1}\times\frac{\sqrt{7}+1}{\sqrt{7}+1}$

$$\rightarrow \frac{\sqrt{7}+1}{\left(\sqrt{7}-1\right)\left(\sqrt{7}+1\right)} \rightarrow \frac{\sqrt{7}+1}{\sqrt{7}\sqrt{7}+\sqrt{7}-\sqrt{7}-1} \rightarrow \frac{\sqrt{7}+1}{7+\cancel{\sqrt{7}}-\cancel{\sqrt{7}}-1} \rightarrow \frac{\sqrt{7}+1}{6}$$

Notice how the surd part always cancels out – difference of two squares,

and the two surds multiply together to give a whole number.

You are using: $(a+b)(a-b) \rightarrow a^2-b^2$